APPLICATION OF PRINCIPAL COMPONENT ANALYSIS TO SIMULTANEOUS SEISMIC INVERSION

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ABSTRACT

The Principal component analysis (PCA) uses an orthogonal linear transformation to project the model space with large dimensionality to a small subsurface dimension by taking advantage of strong correlation between models. In this paper, we apply PCA as an efficient parameterization tool for simultaneous seismic inversion. We start with a large number of training images sampled from the prior probability distribution. Principal components are then calculated from the covariance matrix derived from these training images. Since the number of principal components (PCs) is much smaller than the original number of model parameters, new images in posterior probability distribution can be efficiently reconstructed by only updating the weights of the PC. In this paper, we applied PCA based VFSA to both post-stack and pre-stack seismic data to invert for 2D impedance profiles. The results from simultaneous inversion demonstrate better lateral continuity than a trace-by-trace inversion using the same optimization tool. The same workflow can also be applied to 3D seismic volumes.

INTRODUCTION

Lateral continuity plays an important role in stratigraphic interpretation of a depositional environment. However, traditional 1D trace-based seismic inversion is generally unable to maintain lateral continuity. Simultaneous inversion of traces from all surface locations along a 2D line or a 3D volume involves optimization of a function with a large number of variables using a large data volume, which is a computationally challenging task. Therefore, efficient model parameterization methods are needed for reduction of computation cost and convergence of model parameters. One popular method employed in reservoir modeling uses pilot points (e.g., Long et al., 2009); this is essentially an up-scaling method using representative cells at only a few sparse locations (pilot points). However, it is not trivial to choose the locations of these pilot points and to recover the rock properties at other locations accurately. To circumvent these problems, we investigate dimension reduction in terms of an orthogonal transformation, called principal component analysis (PCA).

Principal component analysis (PCA) has been successfully applied as an efficient parameterization tool in image recognition, compression (Kim, 2002), reservoir modeling (Echeverria1 and Mukerji, 2009) and history matching (Chen et al., 2012). Given a set of training images, PCA linearly transfer them into a set of uncorrelated principal components. The number of these principal components is usually much smaller than that of the model parameters, but they are sufficient enough to represent the diversity of the model space and are able to reconstruct it with a small amount of error.

Application of PCA in seismic inversion brings a new concept to the inversion process. At first, thousands of different training images containing 2D elastic properties are simulated in order to cover the maximum uncertainty range of model prior distribution. Then their principal components (PCs) are calculated from the covariance matrix of training images. Based on the major PCs, which usually take over 80% of the total energy from the prior model space, we are able to sample images in posterior distribution by updating the weights of PCs.

The PCA based simultaneous inversion of a 2D seismic line is tested in this paper with both post-stack and pre-stack seismic data sets. By comparing the inverted impedance profile with the result from trace by trace inversion, better lateral continuity is demonstrated from simultaneous inversion using the same optimization tool (VFSA).

Methods

- **Principal component analysis (PCA)**

The PCA uses a linear transformation to project a large set of prior models into a small number of orthogonal basis or PCs with the goal of minimum average Euclidean reconstruction error $\sum_{i=1}^{N}\|\mathbf{m_i}-\hat{\mathbf{m}}_\mathbf{i}\|_2^2$ (Echeverria and Mukerji, 2009), where $\mathbf{m_i}$ is the original observation and $\hat{\mathbf{m}}_\mathbf{i}$ is the reconstructed one. These PCs can be calculated by identifying the eigenvectors of the covariance matrix derived from the prior models. The variance of each eigenvector can be presented by their corresponding eigenvalues. If we arrange the eigenvectors with their corresponding decreasing eigenvalues, we notice that about 40% of the eigenvectors usually contain more than 80% of the total energy, which is the summation of all the eigenvalues. Therefore, we are able to reconstruct the models based on these 40% of eigenvectors with little loss in resolution.

Let us, for example, consider PCA on image reconstruction: given M number of training images with N number of cells (N = rows of image * columns of image), the workflow of PCA can be summarized as follows (Kim, 2002):

1: vectorization of each image into a column of length *N* and storing M images into a matrix of size N * M:

$$\mathbf{m_i} = [p_{1\ldots}p_N]^T, i = 1,\ldots,M \quad . \tag{1}$$

2: subtraction of the mean image m_0 from each image vector:

$$\mathbf{m_0} = \frac{1}{M}\sum_{i=1}^{M}\mathbf{m_i},\ \mathbf{q_i} = \mathbf{m_i} - \mathbf{m_0} \quad . \tag{2}$$

3: calculation of the eigenvalues λ_i and eigenvectors e_i of the covariance matrix **C**

$$\mathbf{C} = (\mathbf{m_i} - \mathbf{m_0})(\mathbf{m_i} - \mathbf{m_0})^T \quad . \tag{3}$$

4: Sorting eigenvectors $\mathbf{e_k}$ from high to low according to their corresponding eigenvalues. Cutting off the rest of the eigenvectors when the summation of the eigenvalues from the major eigenvectors reaches most part of the total energy (in this paper, the threshold value: *t* is set as 80%).

$$\sum_{i=1}^{k}\lambda_i > t\sum\lambda_i \quad . \tag{4}$$

5: Reconstruction of the image using the coordinates c_i defined by the selected eigenvectors

$$\hat{\mathbf{m}} = (\mathbf{e_1} \quad \ldots \quad \mathbf{e_k})(c_1 \quad \ldots \quad c_k)^T + \mathbf{m_0} \quad . \tag{5}$$

- **Workflow of simultaneous seismic inversion using PCA and VFSA**

The workflow of the PCA based simultaneous inversion using an example of a 2D seismic line is illustrated in Figure 1, which can be also applied to 3D seismic volumes. At first, 1000 training images of elastic properties along the 2D line are simulated using 10 parallel VFSA trace based inversion with different starting temperatures ranging from 10^o to 100^o. Fractal initial models or fGn (Srivastava and Sen, 2009, 2010) is employed to extend the frequency band of the impedance model. Then, PCA is investigated by selecting the most important eigenvectors according to the magnitude of their corresponding eigenvalues. We observe that more than 80% of the total energy is carried by the first 200 eigenvectors (Figure 2). Based on these 200 eigenvectors, the elastic model can be updated through randomly perturbing weights using VFSA until significant small misfits are obtained.

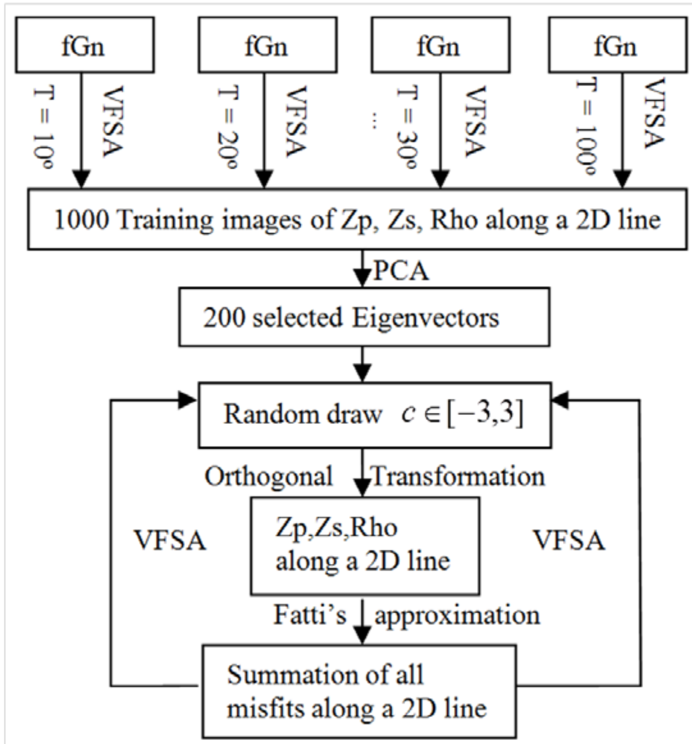

Figure 1. Workflow of PCA based simultaneous inversion along a 2D line using VFSA.

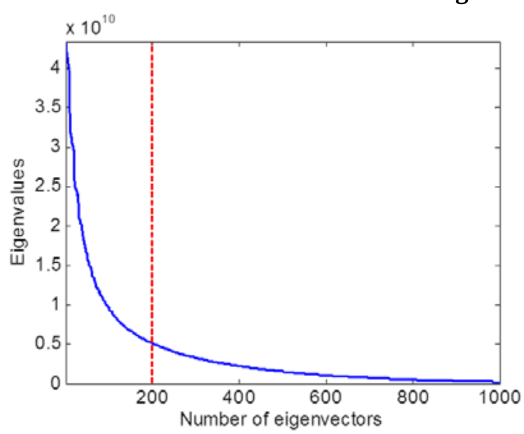

Figure 2. energy plot of spaces constructed by 1000 training images for pre-stack inversion

- ## Forward modeling and objective function

Three steps are involved in the forward modeling. At first, orthogonal transformation is applied to transfer the subsurface constructed from the selected PCs to the real elastic impedance space with coefficients $c_{,,...,}$ c_k, which are initially sampled from a Normal distribution and the average of impedance $\mathbf{Zp_0}$ (Equation 6) from the training images. Then reflection coefficient is calculated for each layer contact (eqn. 7 for post-stack and Equation 8 for pre-stack seismic using Fatti's approximation). At the end, synthetic seismic traces are generated by convolution of the reflection coefficient with the wavelet **w** (Equation 9)**,** which is extracted from the seismic data using Hampson and Russell software (HRS).

$$\mathbf{Zp} = (\mathbf{e_1} \quad \dots \quad \mathbf{e_k})(c_1 \quad \dots \quad c_k)^T + \mathbf{Zp_0}, \tag{6}$$

$$R = \frac{Zp_{i+1} - Zp_i}{Zp_{i+1} + Zp_i}, \tag{7}$$

$$R_{\mathrm{pp}}(\theta) = (1+\tan^2(\theta))[\ln Zp_{i+1} - \ln Zp_i]/2 - 4\left(\frac{Zs_i}{Zp_i}\right)^2 \sin^2\theta \cdot [\ln Zs_{i+1} - \ln Zs_i] - \left[\frac{\tan^2(\theta)}{2} - 2\left(\frac{Zs_i}{Zp_i}\right)^2 \sin^2\theta\right]\frac{\Delta\rho}{\rho}, \tag{8}$$

$$\mathbf{S} = \mathbf{R} \otimes \mathbf{w}. \tag{9}$$

The objective functions for both post-stack and pre-stack inversion are normalized L_2 norms measuring a misfit between the synthetic and observed seismic for all traces (Equation 10).

$$d = \frac{\sum_{i,j} (\mathbf{S_{obs}} - \mathbf{S_{syn}})^2}{\left(\sum_{i,j} (\mathbf{S_{obs}} + \mathbf{S_{syn}})^2 + \sum_{i,j} (\mathbf{S_{obs}} - \mathbf{S_{syn}})^2\right)}. \tag{10}$$

- **Seismic data sets**

To test the workflow of 2D simultaneous inversion, post-stack seismic (Figure 3) and pre-stack seismic angle gathers ranging from 3 degree to 27 degree (Figure 4 and Figure 5) given as demonstration data sets in Hampson and Russell software (HRS) STRATA module along two different lines are investigated. Before inversion, well tie, wavelet extraction and angle gather generation were carried out using HRS. For the pre-stack data, we have one seismic tied well with P impedance log (without shear wave velocity) located at the xline 71 in Figure 4 (corresponding to the trace number of 40 in the Figure 5) for quality control.

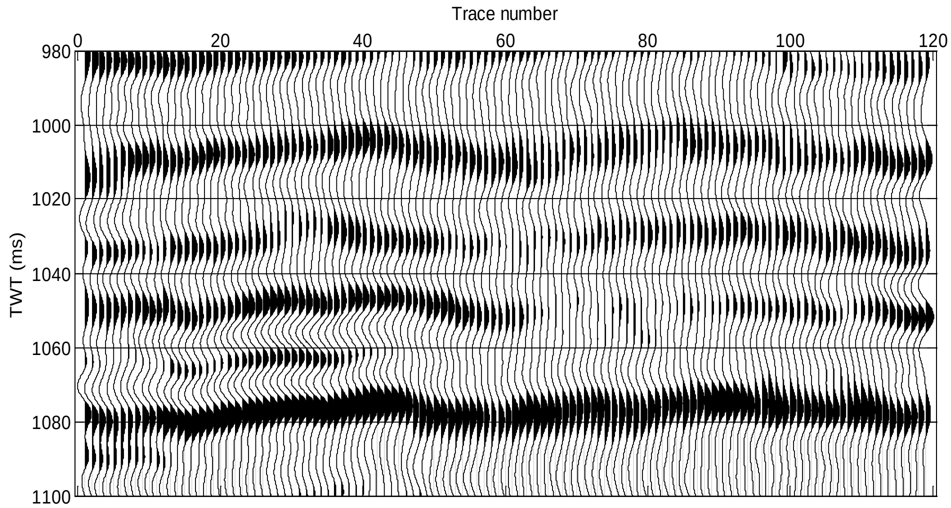

Figure 3. post-stack seismic data along a 2D line from HRS demo data.

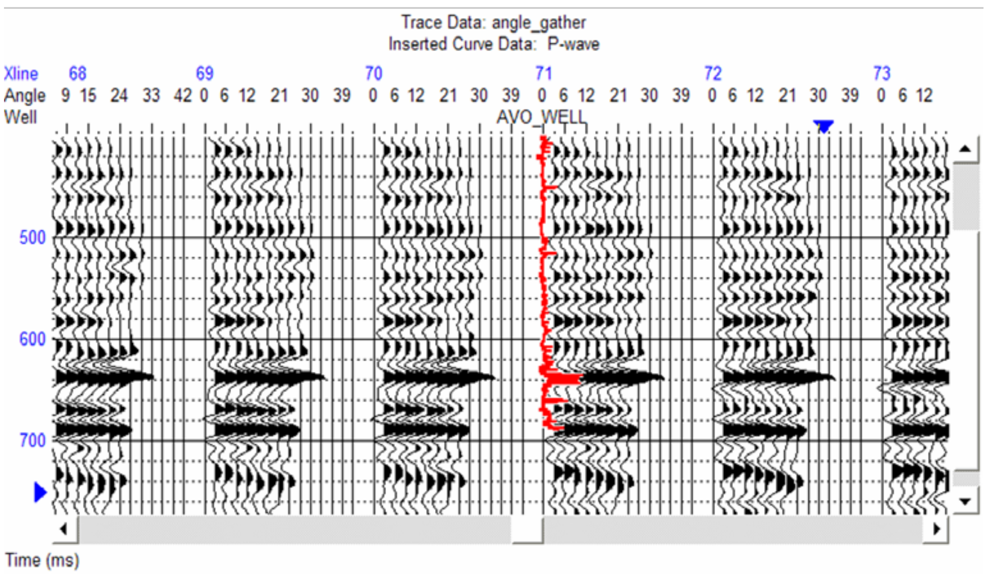

Figure 4. pre-stack angle gather along a 2D line from HRS demo data.

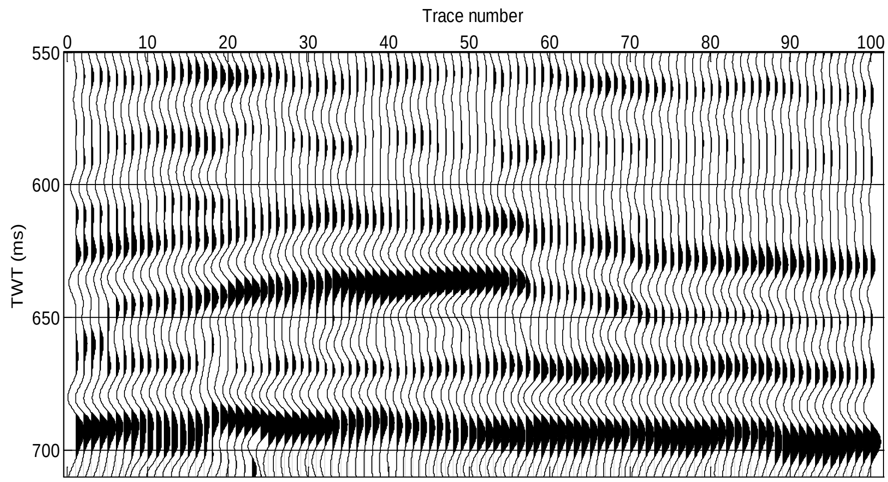

Figure 5. pre-stack seismic data with the angle of 24 degree along a 2D line from HRS demo data. The well log of P impedance marked as red curve in Figure 4 is located at the trace number of 40 in the Figure 5.

Results

- **Inversion of post-stack seismic data**

The P impedance along the 2D line from trace by trace inversion using VFSA is compared with the P impedance from simultaneous inversion using PCA based VFSA (Figure 6). The synthetic post-stack seismic derived from PCA based VFSA and its residuals are plotted in Figure 7.

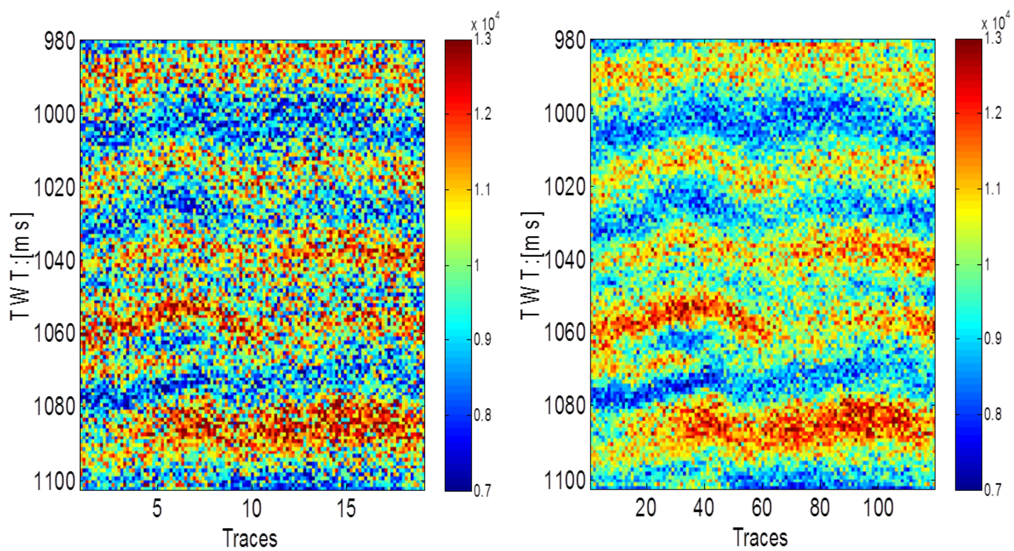

Figure 6. P impedance derived from trace by trace inversion using VFSA (left) and P impedance derived from simultaneous inversion using PCA based VFSA (right).

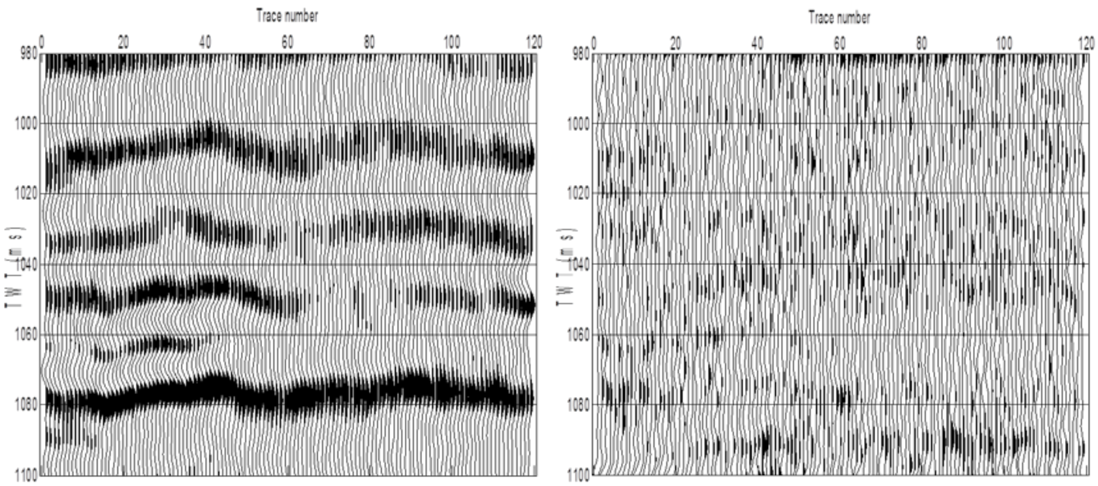

Figure 7. synthetic seismic derived from the best fit impedance model of simultaneous inversion using PCA based VFSA (left) and its residuals compared with the observed post-stack data (right).

- **Inversion of pre-stack seismic data**

Similar process of the post-stack data inversion is applied now on another 2D line of pre-stack seismic data (Equation 8). The P-, S-impedance and P/S impedance ratio derived from simultaneous inversion are compared with these derived from trace by trace inversion (Figure 8, 9 and 10). The synthetic pre-stack seismic at angle of 24 degree derived from PCA based VFSA and its residuals are plotted in Figure 11.

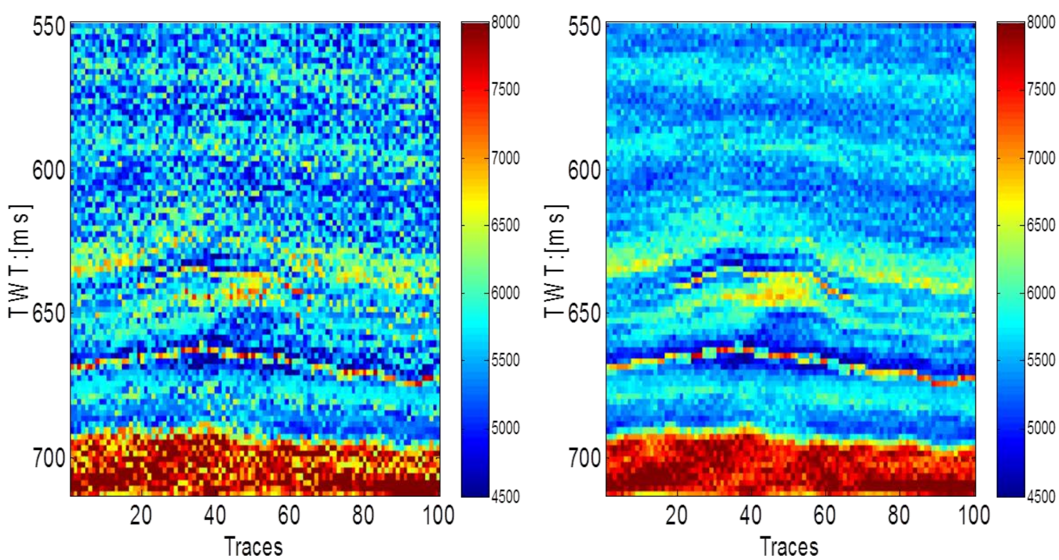

Figure 8. P impedance from trace by trace inversion using VFSA (left) and P impedance from simultaneous inversion using PCA based VFSA (right).

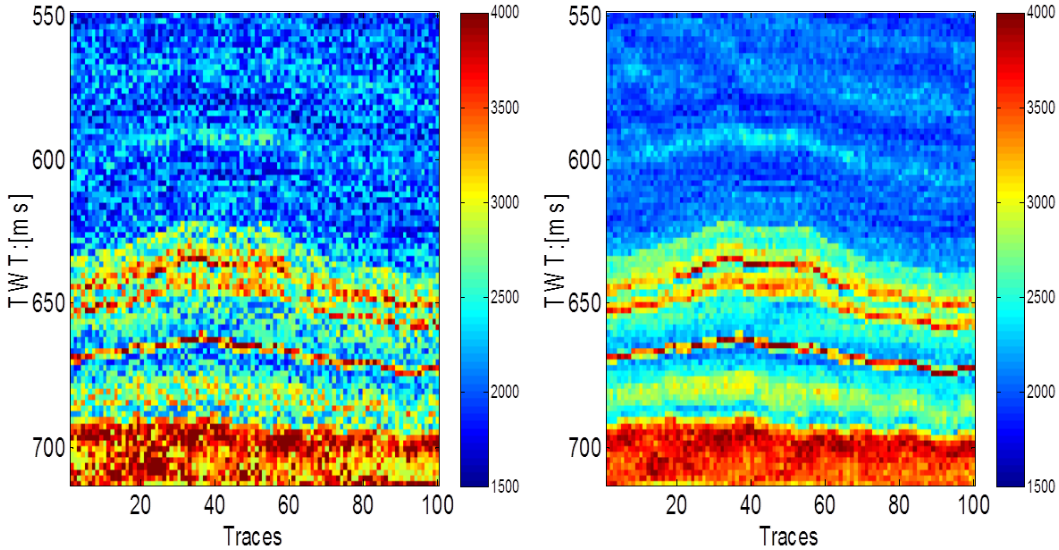

Figure 9. S impedance from trace by trace inversion using VFSA (left) and P impedance from simultaneous inversion using PCA based VFSA (right).

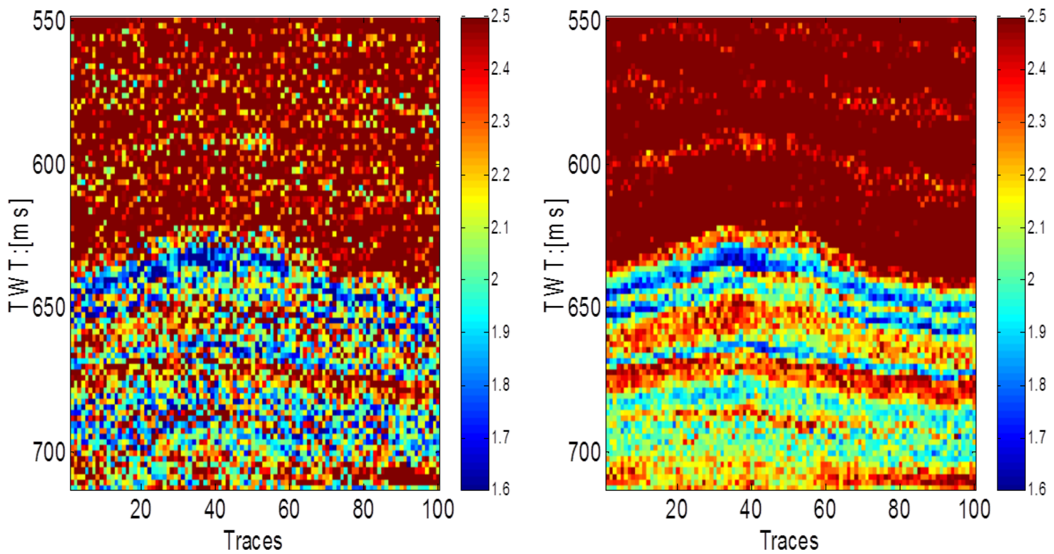

Figure 10. P/ S impedance ratio from trace by trace inversion using VFSA (left) and P/S impedance ratio from simultaneous inversion using PCA based VFSA (right).

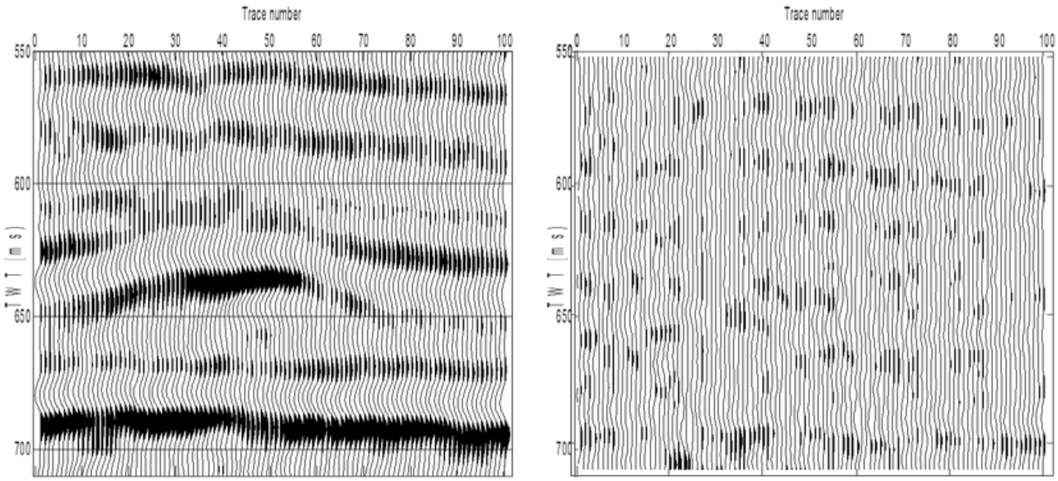

Figure 11. synthetic pre-stack seismic at angle of 24 degree derived from the best fit impedance model of simultaneous inversion using PCA based VFSA (left) and its residuals compared with the observed pre-stack seismic at angle of 24 degree

DISCUSSIONS AND CONCLUSIONS

In this paper, we investigated simultaneous inversion of one hundred traces along 2D lines using PCA as an efficient parameterization tool. The number of the model parameters can be significantly reduced while the residuals of synthetic seismic remain small. Comparison of trace by trace inversion and simultaneous inversion of 2D line using the same optimization tool (VFSA) shows that we are able to obtain better lateral continuity from simultaneous inversion in the examples of both post-stack and pre-stack data.

The advantage of PCA is use of a small number of eigenvectors to represent the model space by considering correlation between model parameters. The purpose of application PCA in simultaneous inversion of seismic lines is to reconstruct the posterior elastic impedance model by simply updating the weights for these eigenvectors, which is much more efficient than directly updating the elastic impedance at each layer and each trace. The critical factor is to generate the prior model space. To address this issue, multiple VFSAs with different starting temperatures are applied at first in order to generate a large number of training images, which are able to approximate the posterior distribution. Furthermore, fractal initial models are applied to extend the frequency band of the model space for higher resolution imaging.

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